Trigonometric Identities and Formulas

Some basic formulas and graphs

$$\sin(-x) = -\sin(x)$$
$$\cos(-x) = \cos(x)$$
$$\tan(x) = \frac{\sin(x)}{\cos(x)}$$

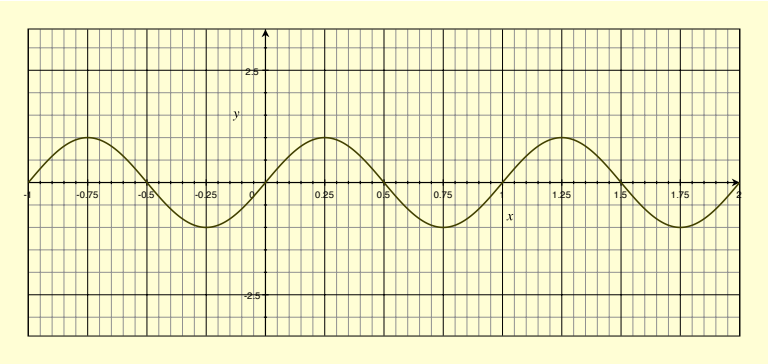


Figure 1. $y = \sin(2\pi x)$

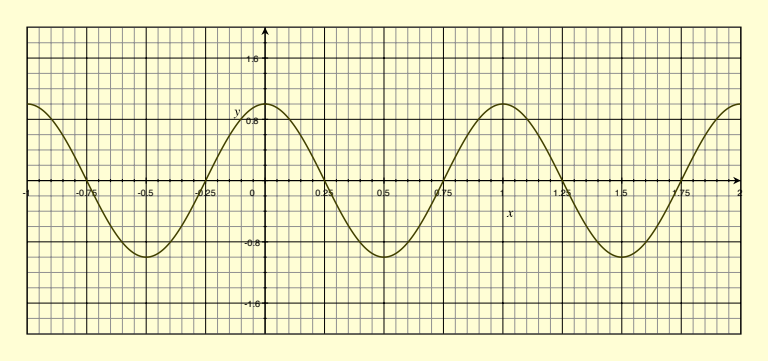


Figure 2. $y = \cos(2\pi x)$

	θ				
	0	30	45	60	90
$\sin\theta$	0	½	$\sqrt{2}/2$	$\sqrt{3}/2$	1
$\cos\theta$	1	$\sqrt{3}/2$	$\sqrt{2}/2$	½	0
$\tan\theta$	0	$\sqrt{3}/3$	1	$\sqrt{3}$	∞

Addition identities

$$\begin{aligned}
\sin(x \pm y) &= \sin(x)\cos(y) \pm \cos(x)\sin(y) \\
\cos(x \pm y) &= \cos(x)\cos(y) \mp \sin(x)\sin(y) \\
\sin(x) \pm \sin(y) &= 2\cos[½(x \mp y)]\sin[½(x \pm y)] \\
\cos(x) + \cos(y) &= 2\cos[½(x - y)]\cos[½(x + y)] \\
\cos(x) - \cos(y) &= -2\sin[½(x + y)]\sin[½(x - y)]
\end{aligned}$$

Product identities

$$\begin{aligned}
\sin(x)\cos(y) &= ½\left[\sin(x + y) + \sin(x - y)\right] \\
\cos(x)\cos(y) &= ½\left[\cos(x + y) + \cos(x - y)\right] \\
\sin(x)\sin(y) &= ½\left[-\cos(x + y) + \cos(x - y)\right]
\end{aligned}$$

Squared identities

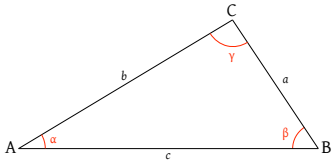
$$\begin{aligned}
\sin^2(x) &= 1 - \cos^2(x) \\
&= ½[1 - \cos(2x)] \\
\cos^2(x) &= 1 - \sin^2(x) \\
&= ½[\cos(2x) + 1]
\end{aligned}$$

Double-angle formulas

$$\begin{aligned}
\sin(2x) &= 2\cos(x)\sin(x) \\
\cos(2x) &= 2\cos^2(x) - 1
\end{aligned}$$

Half-angle formulas

$$\begin{aligned}
\sin(x/2) &= \sqrt{½(-\cos(x) + 1)} \\
\cos(x/2) &= \sqrt{½(\cos(x) + 1)}
\end{aligned}$$



Law of cosines

$$c^2 = a^2 + b^2 - 2ab\cos(\gamma)$$

Law of sines

$$\frac{a}{\sin(\alpha)} = \frac{b}{\sin(\beta)} = \frac{c}{\sin(\gamma)}$$

Exponential identities

$$\begin{aligned}
e^{ix} &= \cos(x) + i\sin(x) \\
e^{-ix} &= \cos(x) - i\sin(x)
\end{aligned}$$

Integrals

$$\begin{aligned}
\int \sin(x)dx &= -\cos(x) + k \\
\int \cos(x)dx &= \sin(x) + k \\
\int \tan(x)dx &= -\ln\left[|\cos(x)|\right] + k
\end{aligned}$$

Derivatives

$$\begin{aligned}
\frac{d\sin(x)}{dx} &= \cos(x) \\
\frac{d\cos(x)}{dx} &= -\sin(x) \\
\frac{\tan(x)}{dx} &= \sec^2(x)
\end{aligned}$$

Simple functions, their derivatives, and their integrals

$f(x)$	$\frac{df(x)}{dx}$	$\int f(x)dx$
a	0	$ax + C$
x^n	nx^{n-1}	$\frac{1}{n+1}x^{n+1} + C^*$
$\sin(x)$	$\cos(x)$	$-\cos(x) + C$
$\cos(x)$	$-\sin(x)$	$\sin(x) + C$
$\ln(x)$	$\frac{1}{x}$	$x\ln(x) - x + C$
e^x	e^x	$e^x + C$

* except $n = -1$