Optics of uniaxial substances

In this laboratory exercise, we will explore the optical properties of uniaxial minerals. The uniaxial minerals include members of the hexagonal, trigonal, and tetragonal crystal systems. Each system has one rotation axis that sets it apart from other systems: A_6, A_3, or A_4. The unique axis is, by convention, parallel to the c axis of the crystal. The other axes, a, are equal in length, are perpendicular to the c axis, and are 120° (hexagonal and trigonal) or 90° apart. Some of the most abundant examples of uniaxial minerals are quartz and calcite. Other mineral examples are tourmaline, apatite, rutile, zircon, scapolite, and vesuvianite.

The optical properties of uniaxial (and biaxial) minerals are characterized by the phenomenon of double refraction. This is demonstrated best by calcite: the image of an object as viewed through a crystal of calcite is doubled. One image is called ordinary, and the other is extraordinary. Light traveling through a uniaxial crystal is split into two waves, each perpendicularly polarized to the other. This occurs because in uniaxial crystals, light with a polarization direction parallel to the c axis has a different speed from that of light polarized in a direction perpendicular to the c axis.

Uniaxial minerals under the petrographic microscope

Double Refraction. Uniaxial and biaxial crystals have the ability to split light into two rays, each polarized perpendicularly to the other. This a result of the lower symmetry and unequal packing of electrons along the crystallographic directions. In hexagonal and tetragonal crystals, there is one distinct crystallographic axis, the 3-fold, 6-fold, or 4-fold rotation axis.

To see the effect of the unique crystallographic axis on the propagation of light through uniaxial crystals, try this experiment. Draw a black dot on a piece of paper. Set a cleavage rhomb of calcite on top, and presto, there are now two dots! This is called double refraction. One ray is called "o" (or ω) for ordinary, and the other is called "e" (or ϵ) for extraordinary. Rotate the cleavage rhomb. What happens? Which do you think is the ordinary and which the extraordinary ray?

There are two pieces of polaroid film available. Play with them to see the effect on intensity as you rotate the polarizers from parallel to crossed. Identify the privileged direction of a sheet of polaroid by observing the change in intensity of light reflected off a surface as you rotate the sheet. Remember Brewster's law, and you shouldn't have any trouble finding the polarization direction.

Take one of the polarizers and view the double refraction of calcite through it. Rotate the polarizer and see what happens to the dots. The polarization direction of the o wave is perpendicular to the plane containing the c axis and the wave normal. The e wave is polarized in the direction perpendicular to the wave normal that lies in the plane containing the c axis. Can you identify the c axis of this calcite rhomb? Make a sketch showing the cleavage rhomb, the c axis, the o and e rays, and the direction of polarization of the rays. Does the e ray obey Snell's law?

The optical properties of uniaxial materials can be represented by an ellipsoid of revolution, called the indicatrix, with the rotation axis representing the extraordinary index of refraction n_e and the circular cross section representing the ordinary index n_o. If $n_e > n_o$, the ellipsoid is prolate, and the mineral is said to be uniaxial positive. If $n_e < n_o$, the ellipsoid is oblate, and the mineral is uniaxial negative. Examples of these ellipsoids can be found in the hand-outs on

the uniaxial indicatrix, which accompany this lab.

All central sections of the ellipsoid are ellipses with one axis equal to n_o (why?). Imagine a plane perpendicular to the light path traveling through the stage. This is the wave front. It is parallel to the microscope stage and perpendicular to the wave normal. Now imagine this plane as a central section through the indicatrix of a randomly oriented mineral lying on the stage. The axes of the ellipse are the vibration directions of the light waves traveling through the mineral. The one vibrating parallel to n_o is the ordinary ray; the other is the extraordinary ray. Prove to yourself that the ordinary-ray vibration direction is perpendicular to the c axis and that the extraordinary-ray vibration direction vibrates in the plane containing the wave normal and the c axis.

A principal section of the ellipsoid is formed in the plane containing the wave normal, which is vertical on the microscope stage, and the c axis of the mineral. This principal section is an ellipse with semimajor and semiminor axes of n_o and n_e. All pertinent directions associated with the light paths through uniaxial minerals can be seen on this section. The ellipse contains the principal indices of refraction, the indices of refraction observed for the orientation of the mineral with respect to the stage, the wave normal, the orientation of the stage, and the extraordinary ray vibration direction. The ordinary-ray vibration direction is perpendicular to the section.

Relief. Make a grain mount of calcite powder in oil with an index of refraction of 1.65. Observe the mount under the microscope in plane polarized light and rotate the stage. You should see a grain just about disappear in one orientation and then stand out with high negative relief when you rotate the stage 90°. Check the Becke lines out to prove this.

For uniaxial minerals, you can always measure the ordinary index of refraction using this type of grain mount method. All you have to do is rotate the grain so that the polarization direction of the light from the substage is perpendicular to the c axis. Then there is no component of the incident light vibrating parallel to the plane containing the c axis, and no extraordinary ray will be formed.

Similarly, if you could orient the grain so that the incident light is polarized parallel to the c axis, you could measure n_e. For most grains, this would be impossible without the aid of a device that could rotate the grain about more than one axis—spindle stages and universal stages can do this, but not the simple petrographic stage. For most grains, the best you could do would be to rotate the plane containing the c axis parallel to the polarization direction. Then you would measure an index of refraction intermediate between n_o and n_e.

Look at the thin section of siderite (slide 109) in plane polarized light. Rotate the stage of the microscope and observe the change in relief of the carbonate mineral. Siderite seems to wink at you as you rotate the stage. This happens because the two indices of refraction for siderite, $n_o = 1.88$, $n_e = 1.64$, have a great difference. This difference is called birefringence, $\Delta = 0.24$. The o ray is considerably retarded relative to the e ray, and the degree of retardation, or wavelength difference, gives the color you see when you cross the nicols.

Birefringence. Birefringence in uniaxial crystals is defined as the difference between the indices of refraction for the o and e waves.

On the microscope, the entering light is plane polarized. If the polarization direction is parallel to one of the privileged directions of the crystal, then only one wave travels through the crystal. The polarization of the wave is the same as that of the entering wave.

In all other orientations, the entering light is split into ordinary and extraordinary waves, even though the light that enters the crystal is plane polarized. This entering light is split into

two components, o and e, which have some proportion of the original amplitude.

As the light exits the crystal, the o and e waves, which are polarized in perpendicular directions, interfere with one another. The result, in general, is an elliptically polarized wave. When the analyzer is placed in the optical path, the component of the elliptically polarized wave parallel to the analyzer is transmitted. The form of the elliptically polarized wave depends on the phase difference between the two exiting waves. While traveling through the crystal, the o and e waves have different wavelengths. This is because there are two different indices of refraction for these waves. Once they exit, the wavelengths become the same (why?).

The elliptical polarization depends on the difference between the number of wavelengths each wave traveled while in the crystal. This difference is called the retardation. The retardation can be determined by considering a crystal with a thickness x, a large index of refraction n_{slow}, and a small index of refraction n_{fast}. Follow the leading edge of a wave as it enters the crystal. It is split into slow and fast waves, according to the large and small indices of refraction. When the fast wave emerges, the slow one is still within the crystal. When the slow wave emerges, the fast one has traveled a distance $x + \delta x$. So in the time it takes for the slow wave to emerge, the fast wave is ahead by δx. This time is

$$x/u_{\text{slow}} = x/u_{\text{fast}} + \delta x/c$$

Substituting the indices of refraction, $c/u = n$, and canceling the common factor c, the relation is

$$xn_{\text{large}} = xn_{\text{small}} + \delta x$$

—remember that the index of refraction of air is ~ 1. Solving this expression for the path difference,

$$\delta x = x(n_{\text{large}} - n_{\text{small}})$$

This is the retardation, and it is given in nm. The relation shows, as it intuitively should, that the path difference depends both on the difference in refractive indices and the thickness of the crystal. If the retardation is an integral number of wavelengths, the exiting waves will be in phase, the resulting elliptically polarized wave is actually a plane-polarized wave, the plane of polarization is parallel to the polarization direction of the entering wave, and no light will pass through the analyzer. If the retardation is an integral number of half wavelengths [(2n+1)/2], the result is a wave plane polarized perpendicular to the entering wave, and all the light passes through the analyzer. If the retardation is an integral number of quarter wavelengths, the result is a circularly polarized wave (remember the homework problem). Any other retardation results in elliptically polarized light.

The reason colors are seen when viewing birefringent crystals through the analyzer is that the incident light is white. The path difference for green light is different from that for red light. Each wavelength will be passed through the analyzer with an intensity that depends on its retardation. The color that is seen is the result of the sum of the intensities of all the wavelengths. Thus crystals with a retardation of one wavelength (called first order) for 550 nm (green) will appear reddish violet. This is because the intensity of 550 nm light passing through the analyzer is zero, but the other wavelengths, both shorter and longer, are passed through to some degree. The result is the observed color reddish violet, or purple. The birefringence chart shows the colors observed in crossed nocols for different values of retardation. Note the cyclic nature of the color patterns. Each cycle is called an order.

Standard thin sections are 30μm thick. Thus, the interference colors observed in thin section are a measure of the birefringence, because the thickness is constant. Any mineral can

have a range in observed interference colors because of the effects of orientation of the mineral with respect to the microscope stage. For example, if the c-axis of quartz is lying parallel to the stage, a pale yellow color will be seen. If the c-axis is perpendicular to the stage, the crystal will appear to be isotropic and the birefringence will be 0 (why?)! The birefringences indicated on the color chart for the different minerals are the maximum birefringences seen. These values are given by the difference between the smallest and largest principal refractive index. This is true even for biaxial minerals.

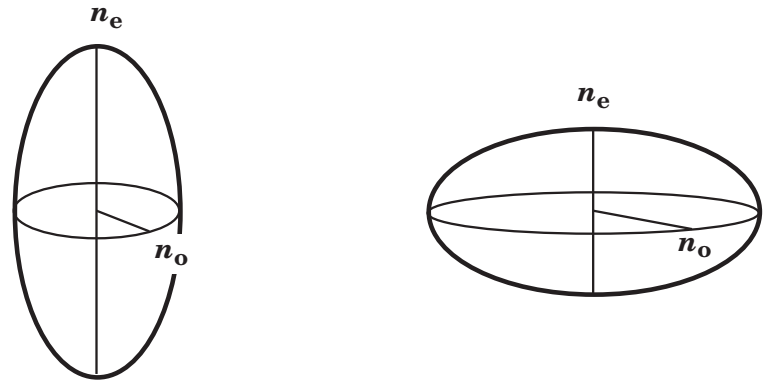
Elongation. There are several characteristics of minerals that you can observe under the microscope that aid in the mineral's identification. Obvious ones include color, relief, fractures or cleavage, shape. The optical class characterizes the symmetry. Is the mineral isotropic, uniaxial, or biaxial (yet to come)? We are concerned with uniaxial minerals, here. Other optical features of distinction include optic sign and birefringence. It is possible to tell whether a mineral is uniaxial positive or negative if the mineral has a characteristic elongate shape. Tourmaline, for example, is almost always elongate. This long dimension, in most cases, is the direction of the c-axis. If only there were some way of determining whether the light vibrating in the long direction is slow or fast, we would know what the optic sign is. There is a way! All you have to do is insert something in the optical path with known optical properties and see what the net effect is. The something is called an accessory plate. You have two, a plate of gypsum with a retardation of 1λ and a plate of mica with a retardation of $1/4\lambda$. In both plates, the slow direction is oriented NE–SW. All you have to do is rotate the mineral grain so the long dimension is NE — this will be a direction of maximum brightness (why?) — and insert the plate. If the interference colors increase, the mineral is length slow; if the colors decrease, the mineral is length fast! You should be able to explain why this works.

Pleochroism. One of the properties of minerals that depends on crystallographic direction is color. Anisotropic minerals can absorb light of differing wavelengths in different directions. The reason usually is that electrons absorb the light in transition to higher energy states. In order for this to happen, there must be a change in the dipole moment. This type of change can only occur in specific crystallographic directions, usually parallel to a chain of polyhedra, for example. A good example of pleochroic mineral is tourmaline. Look at tourmaline in plane polarized light. As you rotate the stage, the color darkens from pale to deep blue or green or brown, depending on the variety of tourmaline. Many colored minerals are pleochroic to some degree. The pleochroic character can be a very important diagnostic property, allowing you to identify the mineral.

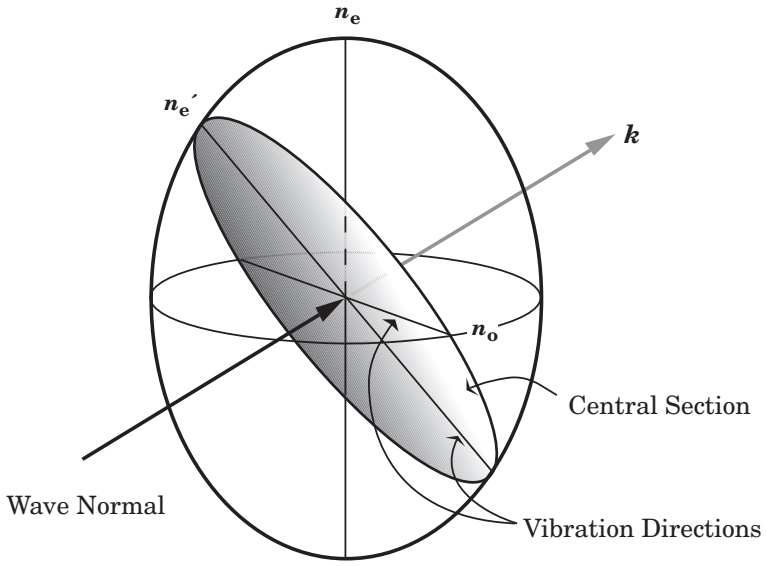
Uniaxial Indicatrix

The difference in index of refraction, which is inversely proportional to speed, is represented geometrically by the ellipsoid called the INDICATRIX. For uniaxial minerals, the ellipsoid is an ellipsoid of revolution with the rotation axis parallel to c, having an index of refraction n_e, and the circular cross section having a radius of n_o. If $n_e > n_o$, the ellipsoid is prolate, and the mineral is said to be optically positive. If $n_e < n_o$, the ellipsoid is oblate, and the mineral is said to be negative.

Prolate Ellipsoid **Oblate Ellipsoid**

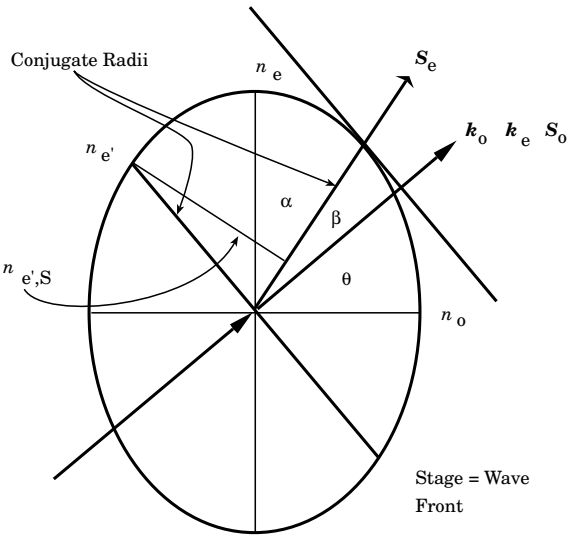


The ordinary and extraordinary waves have polarization directions and indices of refraction given by the central section of the indicatrix that is perpendicular to the wave normal. The central section has a semimajor and semiminor axis; these are the polarization directions, and their lengths give the indices of refraction. The o wave is polarized in the direction perpendicular to the plane containing the c axis and the wave normal. The e wave is polarized in the direction that lies within the plane but is also perpendicular to the wave normal. The index of refraction of the o wave is always n_o; the index of refraction of the e wave, n'_e, is generally between the value of n_o and n_e. In the special case for which the wave normal is $\perp$ c, the e wave has the index n_e. In the special case for which the wave normal is $\parallel$ c, there is no e wave because the central section is the circle with radius n_o, and light behaves as if it were passing through an isotropic mineral. A sketch of the section is shown below.



The light path itself represents the wave normal **k**, and you can think of the plane normal to this direction as the microscope stage.

If you think about it, all the pertinent information for the uniaxial indicatrix can be represented by the principal section defined by the c axis and the wave normal. The vibration direction of the extraordinary wave lies in this plane and that for the ordinary wave is perpendicular to this plane. One feature that can be seen in this type of view is the difference between the ray direction and the wave normal.



In the above drawing, the extraordinary wave front and the extraordinary ray direction are conjugate radii of the ellipse. Conjugate radii have the following properties: they form a parallelogram with sides tangent to the ellipse, the area circumscribed by all sets of conjugate radii is constant, and one radius extends to the point of tangency of the other radius. Knowing the direction of the wave normal with respect to the c-axis and the indices of refraction, it is possible to calculate the value of n'_e and n'_{e_S} and the direction of the extraordinary ray from the geometric properties of an ellipse.

If θ is the angle between a radius of an ellipse and the semimajor axis b, then the equation of the ellipse in polar coordinates is

$$r = \frac{ab}{\sqrt{b^2 \sin^2\theta + a^2 \cos^2\theta}},$$

with a as the semiminor axis. For the case of the principal section of the indicatrix, above, the equation can be rewritten, with the aid of a trigonometric identity, as

$$n'_e = \frac{n_\mathrm{o}}{\sqrt{1 + \left[\left(\frac{n_\mathrm{o}}{n_\mathrm{e}}\right)^2 - 1\right]\cos^2\theta}}.$$

This gives the value of n'_e for any orientation of the wave front with respect to the c axis of the crystal.

The length of the radius conjugate to n'_e, that is, in the extraordinary ray direction, is given by

$$r_{S_\mathrm{o}} = \sqrt{n_\mathrm{o}^2 + n_\mathrm{e}^2 - {n'_\mathrm{e}}^2}.$$

The angle β between the wave normal and the ray direction is given by

$$\beta = \cos^{-1}\left(\frac{n_\mathrm{o} n_\mathrm{e}}{n'_\mathrm{e}} r_{S_\mathrm{o}}\right).$$

The magnitude of the index of refraction for the ray is given by

$$n'_{\mathrm{e}_S} = \frac{n_\mathrm{o} n_\mathrm{e}}{r_{S_\mathrm{o}}}.$$

This is component of n'_e normal to the ray direction. These relations are determined from the properties of conjugate radii.

Exercises.

1. Perform the experiments with the polaroid, the calcite grain mounts, and the siderite thin section, as described above.

2. Observe the cross polarized state through your microscope. If the Nicols are perpendicular, no light should pass through. Now insert the 1λ plate in the light path. What color do you see? Now rotate the analyzer so that the vibration direction is parallel to that of the polarizer. What color do you see? The orders of birefringence are defined on the basis of 550 nm. For light of wavelength 550 nm, retardations ranging from 0 to 550 nm are called first order, retardations ranging from 550 to 1100 nm are called second order, and so on. Your book has a chart of the birefringence "spectrum."

3. What interference color would you observe for a 30-µm-thick section of quartz cut perpendicular to the c axis? What color would you see for a 30-µm-thick section of quartz cut parallel to the c axis if the c axis is oriented 45° to the polarizer? Parallel to the polarizer? What if the section is 40 µm thick? What thickness would have a retardation of 1nm?

4. If a uniaxial crystal has a prominent {010} (or $\{01\bar{1}0\}$) cleavage, what would be the angle between the polarizer direction and the cleavage trace when the crystal is at extinction? This is called the extinction angle.

5. Make an orientation drawing for quartz showing the crystallographic axes, the direction of o and e, and the values of n_o and n_e. Look in an optical mineralogy textbook for examples of orientation diagrams.

6. Examine the thin sections of rock samples that contain examples of uniaxial minerals. Learn to recognize these minerals quickly. Learn to recognize the association of minerals. Does quartz occur in the corundum-bearing sample? Why or why not? Knowing mineral associations is one of the keys to quick mineral identification.

Thin sections: 2 Quartz in porphyry, 20 cancrinite in porphyry, 25 leucite in tephrite, 95 corundum in syenite, 105 apatite in volcanic flow, 107 calcite in marble, 108 dolomite in dolostone, tourmaline.