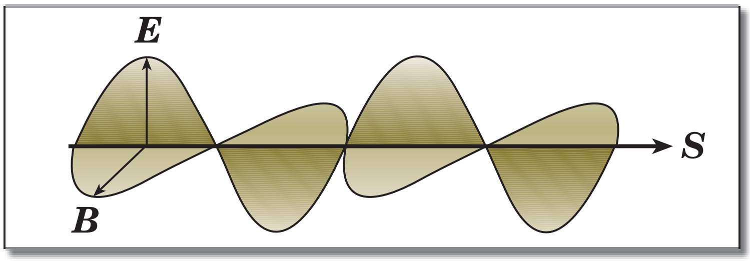
Optics of Isotropic Substances

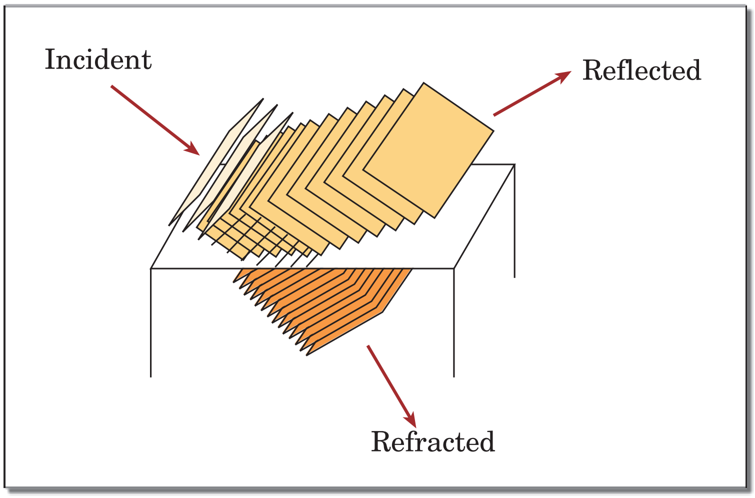
In this lab you will become familiar with the optical properties of isotropic materials. The concepts introduced here are: INDEX OF REFRACTION, SNELL'S LAW, TOTAL INTERNAL REFLECTION, POLARIZED LIGHT, BREWSTERS ANGLE, DISPERSION, and BECKE LINES.

Index of refraction. The index of refraction of a dielectric (non-conducting) substance is defined as $n \equiv c/u$, where c is the speed of light in a vacuum ($\approx$ speed in air) and u is the speed of light in the substance. The value of c is $2.997924562 \times 10^8 \pm 1.1$ m/s. Because this is a maximum value, in accordance with special relativity, the values of indices of refraction are all ≥ 1.0.



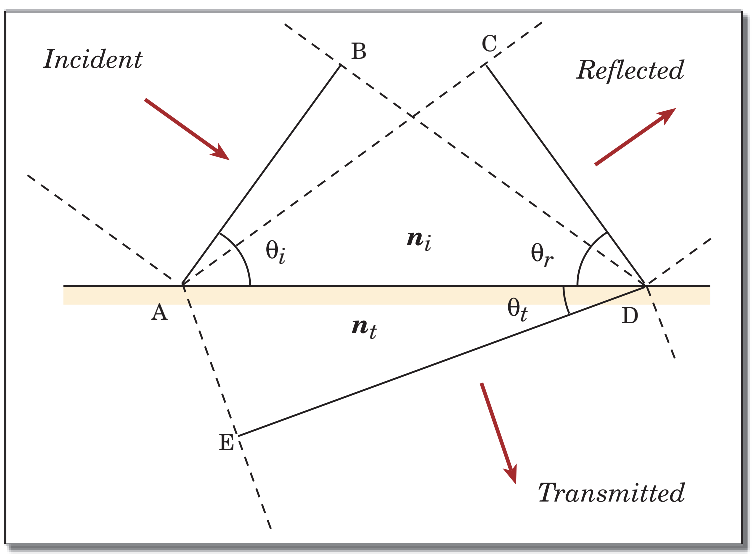
Visible light, as a member of the electromagnetic spectrum, can be visualized as containing electric and magnetic components, as seen in the sketch above. **E** is the electric field vector, and **B** is the magnetic field vector. **S** is called the Poynting vector, which represents the direction of energy transport by the electromagnetic wave. The time-varying electric and magnetic fields, which constitute the light wave, must satisfy Maxwell's equations. This requirement leads to the relation $c = 1/\sqrt{\epsilon_0 \mu_0}$, where ϵ_0 is the electric permitivity of free space and μ_0 is the magnetic permeability of free space. Both are fundamental constants that can be determined by experiment; $\epsilon_0 = 8.85 \times 10^{-12}$ s^2 C^2/m^3 kg; $\mu_0 = 4\pi \times 10^{-7}$ m kg/C^2. In dielectric materials, the permitivity and permeability are related to the fundamental constants by $\mu = K_\mu \mu_0$ and $\epsilon = K_\epsilon \epsilon_0$; K_ϵ is called the dielectric constant. You can easily show that the index of refraction must be $n = K_\mu K_\epsilon$. For all but ferromagnetic materials, $K_\mu \approx 1.0$. Therefore, the index of refraction is the square-root of the dielectric constant.

Reflection and refraction. Linear propagation of light can be represented by a series of planar wave fronts, which at any instant have the same phase. Rays of light are the normals to the wave fronts.



The figure above illustrates plane waves as they impinge on an interface between media with different indices of refraction.

Suppose that each wave front represents the maximum amplitude. The sketch below illustrates that the frequencies of all three waves, incident, reflected, and transmitted, are equal. Each time an incident wave strikes the interface, reflected and transmitted waves are created.



In the time it takes the incident beam to travel from B to D in the above figure, the transmitted wave front travels from A to E and the reflected front travels from A to C. The velocities of the beams are $v_i = c/n_i$, $v_r = c/n_i$, and $v_t = c/n_t$. The n is the index of refraction. If it takes time t for all these wave fronts to move the indicated distances, then the distances are $BD = v_i\ t$, $AE = v_t\ t$, and $AC = v_i\ t$. Note that the distance AD is common to the three right triangles.

Thus, $AD = BD/\sin\theta_i = AC/\sin\theta_r = AE/\sin\theta_t$. Substitute the velocities-times-time for the distances and get

$$\frac{\sin\theta_i}{v_i} = \frac{\sin\theta_r}{v_i} = \frac{\sin\theta_t}{v_t}.$$

Two basic laws come from this relation. The first is the LAW OF REFLECTION:

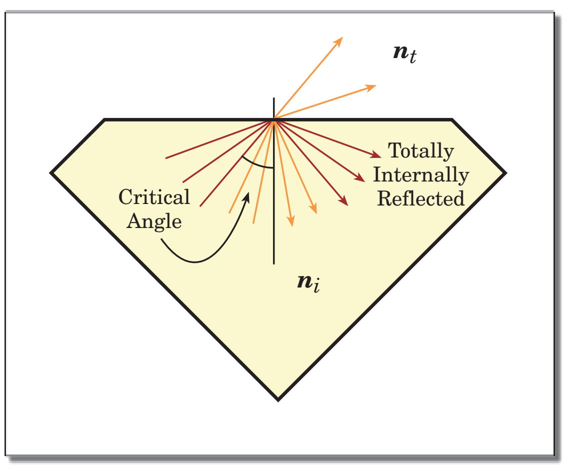
$$\theta_i = \theta_r.$$

This can be seen from the first two terms because the velocity in the medium i is the same for both waves. The second law comes from substituting the indices of refraction for the velocities in the first and third terms above. This gives the LAW OF REFRACTION (or SNELL'S LAW):

$$n_i \sin\theta_i = n_t \sin\theta_t.$$

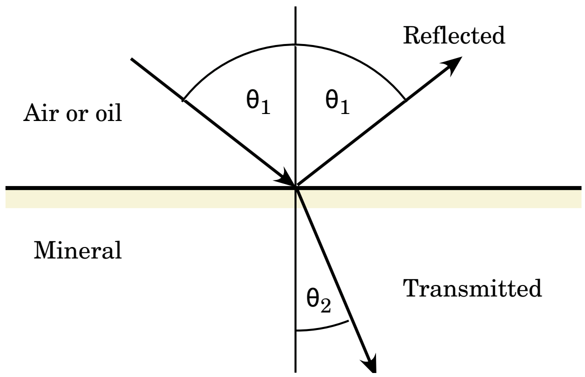
The normal to the incident wave, which we can loosely call the RAY, and the normal to the interface define a plane called the PLANE OF INCIDENCE. Because of the symmetry of the process, the reflected and transmitted rays will also lie in the plane of incidence. We will use the symbol $\mathbf{k}$ to represent the vector normal to the wave front; its magnitude is $2\pi/\lambda$.

If light travels from a medium of low refractive index into a medium of high refractive index, both reflected and transmitted waves are present. However, if light travels from a high-index medium into a low-index one, there is a critical angle of incidence beyond which no light is transmitted across the interface. The result is TOTAL INTERNAL REFLECTION.

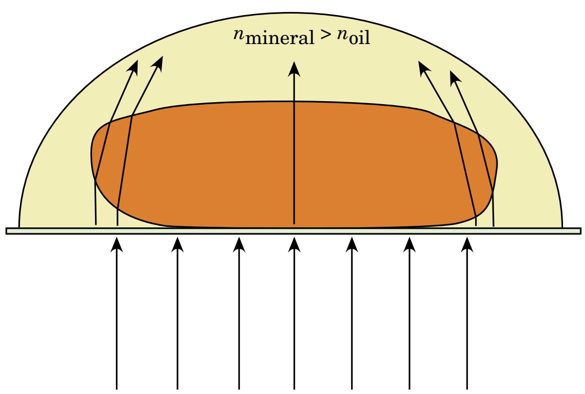


The critical angle can be determined from Snell's Law by noting that the maximum value of the sine is 1, corresponding to an angle of 90°. The critical angle θ_c occurs when the transmitted angle θ_t, is 90° and is given by $\theta_c = \sin^{-1}(n_t/n_i)$. Diamond, for example, has an index of refraction of 2.42, which gives a critical angle of 24.4°. So much of the light in diamond is totally internally reflected that it is quite brilliant!

Becke Lines. As light enters one medium from another, light is refracted according to Snell's law: $n_1 \sin\theta_1 = n_2 \sin\theta_2$.

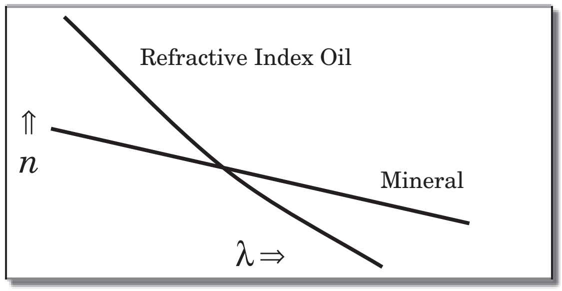


If the indices of refraction of the two media are identical, then the boundary between them is artificial, as far as light is concerned, and the boundary seems to disappear. If a grain of a mineral is immersed in a fluid having the same refractive index, the grain will be invisible. If the indices are different, the mineral stands out from the fluid. This is called relief. The boundary is called Becke's line. Because of the irregularities at the edges of grains, light passing through the grain will be refracted either inward or outward, depending on whether the grain has a higher or lower index than the fluid (see the illustration).



If you defocus the grain by moving the stage downward (or the tube upward), you will see which way the light is refracted by the apparent movement of the Becke lines toward the medium with the higher index. Try closing the iris diaphragm; this helps to see the Becke lines. The method of Becke's line allows you to determine quickly the index of refraction of minerals in index oils.

Dispersion. The index of refraction, it turns out, depends on the frequency of light propagating through the medium. This dependence on frequency, called dispersion, is a natural consequence of the electromagnetic wave nature of light. The dispersive properties of a dielectric can be measured as the difference in indices of refraction at two wavelengths of light, usually a blue and a red wavelength. The index of refraction generally increases with increasing frequency, or, equivalently, with decreasing wavelength. Solid materials commonly have low dispersion; whereas, the oils we use for determining the index of refraction have greater dispersion. This is illustrated below.



It is common to refer to specific wavelengths of light by their Fraunhofer symbols. These represent the spectral lines absent from the solar spectrum because of absorption by gases in the sun's atmosphere. The symbols themselves were arbitrarily assigned by Fraunhofer. Some lines are:

Line	Wavelength (nm)
B	687.0
C	656.3
D_1	589.6
D_2	589.0
E	527.0
b	518.3
F	486.1
G	430.8

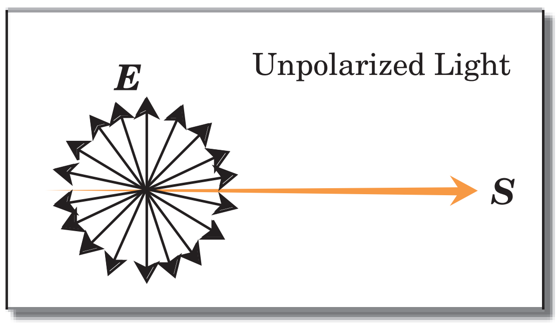
The D lines are due to the absorption by sodium and occur in the yellow part of the spectrum. Indices of refraction are commonly reported as the index for sodium-D light, n_{D}.

The coefficient of dispersion is the difference in refractive indices for light corresponding to the C and F Fraunhofer lines: $\Delta = n_{\mathrm{F}} - n_{\mathrm{C}}$. The dispersion in quartz might be considered typical; $\Delta = 0.0079$, less than 0.5% of the ordinary index of refraction. Diamond has very high dispersion; $\Delta = 0.0574$, $\sim 2\%$ of its nominal index of refraction. The combination of high index of refraction, 2.419, and high dispersion in diamond give it the fiery flashes that make it sparkle.

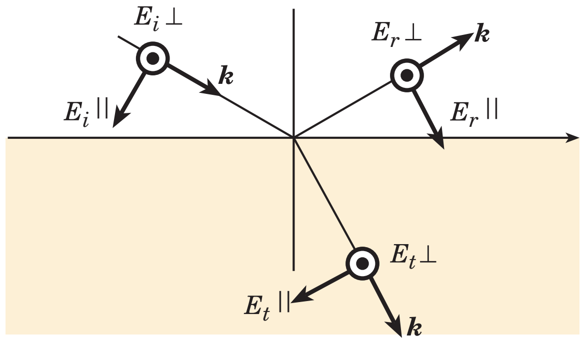
Index oils generally have higher dispersion than solids. For example, nitrotoluene has a coefficient of dispersion of 0.024. Many high-dispersion oils have Δs greater than 0.03 for asbestos identification.

Convince yourself from the above graph that oils have higher indices of refraction than minerals at short wavelengths, and the opposite is true at long wavelengths. In a grain mount in oil under the microscope, then, the long and short wavelengths will be refracted in opposite directions. A bluish Becke line will move away from the grain, a reddish line will move toward the center of the grain, and a yellowish or orangish line will remain at the edge upon defocusing the grain downward. This observation is extremely useful in determining the indices of refraction and the coefficients of dispersion of minerals.

Polarized Light. The first figure in this handout shows an electromagnetic wave with the electric vector oscillating in the z direction, and the magnetic vector vibrating in the y direction. If a wave maintains these orientations, it is said to be plane-polarized. Generally, the electric vector is of concern in discussing the propagation of light, so we need consider this vector only. The magnetic field vector always is perpendicular to **E**, forming a right-handed set with the Poynting vector. In plane polarized light, **E** always lies in a plane.



Most light is unpolarized. Its **E** does not always lie in a plane. There are several ways of producing polarized from unpolarized light. Anisotropic minerals are extremely effective in producing polarized light. Generally, polarized light is created by a filter of polaroid, or similar material. Polaroid has a film of long-chain organic molecules, which are aligned parallel to each other by stretching the film as it is deposited on a plastic substrate. The organic molecule undergoes electronic transitions with energies in the visible region. The molecules can absorb light when **E** is parallel to the chain. If **E** is perpendicular to the chain, no energy can be absorbed, and the light is transmitted by the film. If **E** is oriented at an angle, not 90°, to the chain, then the component that is parallel is absorbed, and the component that is perpendicular is transmitted. No polaroid is a perfect polarizer and some unpolarized light always gets through the filter.



Plane of Incidence

A third way polarized light is generated is by reflection off a surface. Unpolarized incident light on an interface between two media having indices n_i and n_t, can be thought as consisting of components vibrating within the plane of incidence and components vibrating perpendicular to the plane of incidence. Snell's law and the law of reflection describe the angles of refraction and reflection, but not the intensities. We have seen for angles greater than the critical angle, the intensity of the transmitted ray is zero. It turns out that the intensities of the parallel and perpendicular components of the reflected and transmitted rays can be determined from the vector and electromagnetic properties of waves (as can Snell's law and the law of reflection). One of the results of this type of analysis, which can be seen in chapter 4 of Hecht and Zajak, *Optics*, is that the relative magnitude of the parallel component of the reflected ray is given by

$$\left(\frac{E_r}{E_i}\right)_{||} = \frac{\tan(\theta_i - \theta_t)}{\tan(\theta_i + \theta_t)}.$$

When the angle between the reflected and transmitted rays is 90°, then the intensity of the parallel component is 0. At the proper angle of incidence for this relation between reflected and transmitted rays, the reflected ray is perfectly plane polarized perpendicular to the plane of incidence (parallel to the interface between the two media). From Snell's law, the proper incident angle can be determined.

$$\begin{aligned} n_i \sin(\theta_i) &= n_t \sin(\theta_t) \\ n_i \sin(\theta_i) &= n_t \sin(\pi/2 - \theta_i) \\ \theta_i &= \tan^{-1}\left(\frac{n_t}{n_i}\right) \end{aligned}$$

This relation is called Brewster's Law. The angle of Brewster's law is usually symbolized as θ_p. By the way, Brewster is the inventor of the kaleidoscope.

Exercises. You may work in pairs to complete the observations.

1. To get a feeling for Becke lines, look at the prepared slide with grains of fluorite, quartz, beryl, apatite, and garnet mounted in epoxy with $n_{\mathrm{D}} = 1.535$. This is the order of increasing index of refraction. Which grain has an index of refraction closest to the epoxy? Defocus the slide and observe the direction of movement of the Becke line. Convince yourself that fluorite has the lowest index and garnet has the highest. The difference in indices between the epoxy and the grain is the degree of relief. Note that fluorite stands out just as much as garnet. Fluorite is said to have negative relief because its index is less than that of the epoxy; garnet has positive relief because its index is greater. Look at quartz again. Note that the Becke lines are colored. As you defocus the grain, a yellow line moves to the center of the grain, and a blue line moves away from the grain. This phenomenon is dispersion. The index for yellow light is a little bit higher in quartz than in the epoxy, and vice versa for blue light.

2. This is a logic problem that requires you to use the becke line test to observe differences in relief between a mineral and surround medium. Your TA has set up mounts of one mineral in five different immersion oils (labeled A, B, C, D, E). Determine the relative refractive index (n) of the immersion oils and rank them from lowest to highest n. (Hint: Use the becke line test to determine if the oil has greater or lower n than the mineral, and then the relief refine the order of oils of greater and lower than each other.)

3. Look at the thin sections of rocks with isotropic minerals and learn to identify them. You should have thin sections with fluorite in it, with garnet, and with spinel. There aren't very many common isotropic minerals, so identification should be relatively easy.